What do the + and − signs mean in Arithmetic?

In Arithmetic, the + sign has only one meaning: Add. 8 + 3
8 + 3 can only mean add 8 and 3.

In Arithmetic, the − sign has only one meaning: Subtract 8 − 3
8 − 3 can only mean Subtract 8 and 3.

What do the + and − signs mean in Algebra?

In Algebra the plus and minus signs have several meanings.

1. They indicate the direction of the number
   +8 up eight
   −8 down eight

2. When they appear outside of parentheses they act as keep/change operators. That is they tell us either to keep or change the sign of the number inside parentheses
   + (5) = +5 keep the sign inside
   + (−6) = −6 keep the sign inside
   − (5) = −5 change the sign inside
   − (−6) = +6 change the sign inside

3. The minus sign could also mean just what it does in Arithmetic: Subtract.
   (8) − (3) means Subtract

How are signed numbers combined?

If all the sign are the same then add their absolute values and keep the common sign.

-1 + 2 + 3 − 4 = -5
+1 + 2 + 3 − 4 = +5

How are numbers with different signs combined?

If an expression has both positive and negative signs, for example: +1 + 2 + 3 − 4 + 5 − 6

#1 Separate the positives and the negatives and combine each group.

Positives: +1 + 3 + 5 = +9
Negatives: −2 − 4 − 6 = −12

#2 Subtract the absolute values of the two results: 12 − 9 = 3

#3 Use the sign of the number with the larger absolute value (the 12)

+1 + 2 + 3 − 4 + 5 − 6 = −12
Where are signed numbers combined in real life?

Charles, in the process of making a budget, is summarizing his weekly expenses: Rent $140, Cellphone $15, Transportation $10, Food $75, Clothing $15, Leisure Activities $10. He gets $100 from a student loan and earns $150 at his part time job. Let’s combine all these elements of his budget.

\[-140-15-75-15-10 = -265\]
\[+100+150 = +250\]

We now combine the two sums: -265+250 = -15

What does the negative mean?

His income is not enough to cover all his expenses. How will Charles make up the difference?

- More part time work?
- Savings from summer?
- Family assistance?
- Reduce expenses?

How do we subtract in Algebra?

What’s the difference between a dirty dime and a shiny penny?

The word difference means Subtract. The difference between any kind of dime and any penny is 10¢ - 1¢ or 9¢.

(+8) - (+3)

The minus in the middle does mean subtract. However, it is easier if, instead of saying “Subtract +3 from +8, you consider the minus sign as a ‘change operator’ and say “Combine a +8 with the opposite of +3.Every expression can be simplified in three basic steps. Just remember the letters: SSC S=Separate S=Simplify C=Combine

S (+8) | - (+3) Separate into terms
S +8 -3 Simplify each term
C +5 Combine the results
How is an expression simplified if it has both addition and subtraction? For example: \(-9 + (3-7) - (-2) + (-6) - (+10)\)

No matter how complicated an expression in Algebra is, you can ‘uncomplicated’ it if you

#1 Think of the addition signs as “keep the sign operators” and the subtraction signs as “change the sign operators.”

#2 Follow the SSC rule. again SSC means:

S=Separate the expression into terms. A term starts with a plus or minus sign that is NOT in parentheses.

S=Simplify each term so that it has no parentheses and is ‘compressed’ into one signed number.

C=Combine the results.

\[
-9 + (7-3) - (-2) + (-6) - (+10) \\
-9 + (7-3) - (-2) + (-6) - (+10) \\
-9 + (7-3) +2 -6 -10 \\
-9 +4 +2 -6 -10 \\
-9 +4 +2 -6 -10 \\
\]

Separate the negatives and the positives and find their sums:

\(-9-6-10 = -25\)

\(+4+2 = +6\)

Combine these 2 sums:

\(-25+6=-19\)
Adding & Subtracting (Combining) Integers

Good example. The "-" is the direction. It means she withdrew money and the 350 is the magnitude or how much.

There are also some "+" entries. Those are the deposits...right?

Right! But, I notice our friend hasn't done the balance column for some time. Let me show you how to combine signed numbers.

First we combine all the deposits which are positives...then combine the negatives...

\[
\begin{align*}
+300 & \quad -50 \\
+50 & \quad -200 \\
+200 & \quad -350 \\
+550 & \quad -600
\end{align*}
\]

It's easy to combine the numbers when all the signs are the same. This is an example of CASE 1!

**CASE 1**
LIKE SIGNS
Add The Absolutes
Keep The Common Sign

So we combine the resulting numbers as follows.

\[
\begin{align*}
|+550| & \quad |-600| \\
600 & \quad -550 = 50 \\
-50
\end{align*}
\]

As you notice...the two results have opposite signs. To combine them is a scenario for CASE 2!

**CASE 2**
UNLIKE SIGNS
Subtract The Absolutes
Use The Sign Of The Larger

Subtracting the absolute numbers we get a result of 50. Then we must remember to use the sign of the number with the larger absolute which was a minus. This will give us a final answer of -50. Uh Oh...looks like you're a little short in your checking account, I think you better make a quick deposit.
As I mentioned before...I'm on a diet. Can anybody tell me how I'm doing so far?

**MY DIET**

<table>
<thead>
<tr>
<th>WEEK</th>
<th>Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>WEEK 1</td>
<td>-2</td>
</tr>
<tr>
<td>WEEK 2</td>
<td>-5</td>
</tr>
<tr>
<td>WEEK 3</td>
<td>+1</td>
</tr>
<tr>
<td>WEEK 4</td>
<td>-2</td>
</tr>
<tr>
<td>WEEK 5</td>
<td>+3</td>
</tr>
</tbody>
</table>

I'll give it a try. That's as easy as pie!

**LET'S SEE NOW...**

First we combine all the negatives...then all the positives. Finally, we combine the resulting numbers, the -9 and the +4, by subtracting their absolute values and using the sign of the larger, giving us -5.

![Math equation](equation)

DID YOU HAVE TO SAY PIE?

Settle down everybody...let's backtrack a little bit!

**Remember absolute values?**

---

Separate comic panels:

- "DANG YOU MILDRED!
- "Well...that's the way the check bounces.
- "Let's continue...

Hey Professor! You got any diet tips for me?

Yoah...you can start with a shave and a haircut!

---

**GRATEFUL DEAD**
Right... but let's be more precise. The absolute of a positive is...

POSITIVE!

...and the absolute of a negative is...

ALSO POSITIVE!

...and the absolute of zero is...

ZERO?

That's right... the absolute value is ALWAYS POSITIVE or ZERO!

However, the procedure is a little different when you have more than one number between the absolutes... watch!

\[-5+3\]

First we simplify the numbers between the absolutes. -5 + 3 gives us -2. Now, the absolute value of -2 to 2, which is our answer.

Professor... I'm confused!

Are you positive?

He's always in the dark Professor.

IN THE DARK? Say... that gives me an idea.

I don't know if I'm positive or negative! I'm just confused!
CLICK

OK folks... it's time to learn about subtraction

IN THE DARK?

I'm scared!

HEY! Who turned out the lights?

Whoa! Whose hand is that?

HEY... somebody took my good pen!

OK... lights on! Now... if you'll excuse my graffiti, I'd like to make a point.

Subtraction acts like this switch. Think of the switch as a subtraction symbol.

SUBTRACTION SWITCH

+ on + off -

"Think Opposite!"

Look at the light. If it's on (or positive) you flip the switch (subtract) and the light is off (or negative).

And if the light is off or negative and you flip the switch again or subtract, then the light is on or positive.

Never mind the light switch... I think the Professor has flipped.

I heard that wiseguy... here let me make it real clear for you.

Hey everybody, check it out!

To Subtract
A poem by Leon H. Krinsky

Change The Sign, Then Combine
The End

+10 - (-8)

See this subtraction sign. Picture it as that light switch because it will change the sign that follows.

Now we combine.

+10 - (-8)
+10 + 8
+10 + 8 = +18

The answer is +18!
Right...he's a poet and doesn't even know it (he he).

That's quite enough Longfellow. Anyhow, did you see what happened to the parentheses and the sign outside.

I GOT ONE PROFESSOR! Once there was a man from Nantucket...

As we just saw, a negative outside of the parentheses changes the sign that follows. The -8 becomes +8 and we remove the negative sign outside and the parentheses as well.

However, a positive outside of the parentheses DOES NOT change the sign that follows.

$$+(-15)$$

-15

So, in this problem, the -15 remains -15 but, we here also, remove the outside positive sign and the parentheses.

Now...in this example, the first negative sign will change BOTH signs that follow.

The 9 has no sign. That means it's positive so it becomes -9 and the -2 now changes to +2.

$$-(9-2)$$

-9+2

Again, we remove the outside negative sign as well as the parentheses.

Can't we finish the problem Professor?

Psssst... keep it down ALGEBRA BREATH!

Sure...just combine, remember? We come up with -7.

$$-(9-2)$$

-9+2

Now...let's get a little bit more complicated!
Here...let me give you a tip!

Not exactly... first separate the problem into smaller parts or "TERMS."

See... it has 4 terms!

Now... we simplify each term, finally we'll combine the four results.

Hey Professor! You must be real disappointed by that answer!

Why do you say that?

The answer was zero... all that work for NOTHING!

Remember combining? And now we have our answer.
Exercise Set 6

1. Evaluate each expression
   a. + 7 + 8
   b. - 7 - 6
   c. - 12 + 7
   d. + 7 - 12
   e. - 8 + 8
   f. 10 - 11
   g. - 1 - 2 - 3
   h. - 1 - 2 + 3
   i. - 12 subtracted from 10
   j. The sum of -7 and 9
   k. 50 minus 20
   l. 124 decreased by -100
   m. the difference between 80 and -80?

2. Separate into terms then evaluate.
   a. 7 less than x
   b. 7 less than -15
   c. x less than 7
   d. x increased by 7
   e. 7 increased by x
   f. 7 is less than x
   g. 7 less x
   h. the sum of -10 and x
   i. the sum of x and y
   j. the difference between x and y
   k. the difference between y and x

3. Simplify each problem 2 different ways
   a. - (8-11)
   b. - (12-5)
   c. + (7-12)
   d. + (-4+11)

4. Separate into terms, simplify each term then combine the results.
   a. - (7) - (8-10) + (12-1)
   b. 10 - (-8) + (-2) - (8-10)

5. For each example, translate then evaluate
   a. 7 less than 10
   b. 7 less than -15

6. 9. Evaluate
   a. 124 decreased by -100
   b. 50 minus 20
   c. the difference between 80 and -80?
Jokes Set #6

A Math student is flying non-stop from Edmonton to Frankfurt with AirTransat. The scheduled flying time is nine hours.

Some time after taking off, the pilot announces that one engine had to be turned off due to mechanical failure: "Don't worry - we're safe. The only noticeable effect this will have for us is that our total flying time will be ten hours instead of nine."

A few hours into the flight, the pilot informs the passengers that another engine had to be turned off due to mechanical failure: "But don't worry - we're still safe. Only our flying time will go up to twelve hours."

Some time later, a third engine fails and has to be turned off. But the pilot reassures the passengers: "Don't worry - even with one engine, we're still perfectly safe. It just means that it will take sixteen hours total for this plane to arrive in Frankfurt."

The Math student remarks to his fellow passengers: "If the last engine breaks down, too, then we'll be in the air for twenty-four hours altogether!"

Brain Teaser Set #6

Note: This brain teaser must be done IN YOUR HEAD ONLY and NOT using paper and a pen.

Take 1000 and add 40 to it.


What is the total?

Now add another 1000.
### Answers to Exercise Set 6

<table>
<thead>
<tr>
<th>Question</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>1a.</td>
<td>+15 or -8+11=+3</td>
</tr>
<tr>
<td>b.</td>
<td>-13</td>
</tr>
<tr>
<td>c.</td>
<td>-5</td>
</tr>
<tr>
<td>d.</td>
<td>-5</td>
</tr>
<tr>
<td>e.</td>
<td>0</td>
</tr>
<tr>
<td>f.</td>
<td>-1</td>
</tr>
<tr>
<td>g.</td>
<td>-6</td>
</tr>
<tr>
<td>h.</td>
<td>0</td>
</tr>
<tr>
<td>2a.</td>
<td>0</td>
</tr>
<tr>
<td>b.</td>
<td>2</td>
</tr>
<tr>
<td>c.</td>
<td>18</td>
</tr>
<tr>
<td>d.</td>
<td>-2</td>
</tr>
<tr>
<td>e.</td>
<td>-2</td>
</tr>
<tr>
<td>f.</td>
<td>-15</td>
</tr>
<tr>
<td>g.</td>
<td>4</td>
</tr>
<tr>
<td>h.</td>
<td>0</td>
</tr>
<tr>
<td>3a.</td>
<td>-(3)=+3</td>
</tr>
<tr>
<td>i.</td>
<td>10–(-12) or 22</td>
</tr>
<tr>
<td>j.</td>
<td>-7 + (9) or +2</td>
</tr>
<tr>
<td>k.</td>
<td>7+x or +5</td>
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<tr>
<td>4a.</td>
<td>+6</td>
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<tr>
<td>b.</td>
<td>+18</td>
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<tr>
<td>c.</td>
<td>7–x or +9</td>
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<tr>
<td>d.</td>
<td>x+7 or +5</td>
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<td>e.</td>
<td>7+x or +5</td>
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<tr>
<td>5a.</td>
<td>10–7 or +3</td>
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<td>b.</td>
<td>-15–7 or -22</td>
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<tr>
<td>c.</td>
<td>-5+15 or +10</td>
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<tr>
<td>d.</td>
<td>-5+5 or 0</td>
</tr>
<tr>
<td>e.</td>
<td>7+x</td>
</tr>
<tr>
<td>f.</td>
<td>7 &lt; x or 7 &lt; -2</td>
</tr>
<tr>
<td>g.</td>
<td>7–x or +9</td>
</tr>
<tr>
<td>h.</td>
<td>-10+x or -12</td>
</tr>
<tr>
<td>6a.</td>
<td>x–7 or -9</td>
</tr>
<tr>
<td>b.</td>
<td>-15–7 or -22</td>
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<tr>
<td>7a.</td>
<td>No</td>
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<td>c.</td>
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<td>d.</td>
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<td>8a.</td>
<td>-5</td>
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<td>c.</td>
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</tr>
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<td>e.</td>
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<tr>
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</tr>
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<td>9a.</td>
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<td>b.</td>
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<tr>
<td>c.</td>
<td>160</td>
</tr>
</tbody>
</table>

### Brain Teaser #6 Answer

Did you get 5000?

The correct answer is actually 4100.

Don't believe it? Check with your calculator!