An exponent is a shorthand way to show that we are MULTIPLYING with the same number. Suppose that we are multiplying with a 3, four times. That’s 3•3•3•3. Here we can say that 4 threes are 81.

\[ 3^4 = 3 \cdot 3 \cdot 3 \cdot 3 = 81 \]

3 is called the base. 4 is called the exponent.

**Why Is An Exponent Called A Shortcut?**

An exponent is a shorthand way to show that we are MULTIPLYING with the same number. Suppose that we are multiplying with a 3, four times. That’s 3•3•3•3. Here we can say that 4 threes are 81.

**What Are Powers Of Ten?**

Some powers of ten are the numbers in the sequence 10, 100, 1000, 10000, etc.

These numbers are generated by using 10 as a base with exponents (or powers) of 1, 2, 3, 4, 5, etc. The exponent tells you how many zeros follow the 1.

\[ 10^1 = 10 \quad 10^2 = 100 \]
\[ 10^3 = 1000 \quad 10^4 = 10000 \]
\[ 10^5 = 100000 \]

**What Are Factors?**

In a multiplication problem each of the numbers being multiplied is called a factor. For example, 7ab would have 3 factors, 7, a, and b. The entire expression 7ab is called a product.

How many factors are in: 9x²y²?

When expanded it looks like this:

\[ 9 \cdot x \cdot x \cdot y \]  There are 4 factors.
How Are Powers Of Ten Used?

#1
Multiply 365 by 1000. There’s really no need to set up the problem and multiply by with all those zeros.
Just write 365 and attach those 3 zeros that you see in 1000.
365 000 or 365,000 is the answer.

#2
Multiply 24 by $10^5$
Simply write 24 and then attach 5 zeros (the exponent is 5).
24 00000 or 2,400,000

How Do I Estimate the Product When I Multiply?

To estimate a product, round each number to the first digit. The numbers will have zeros at the end. Multiply these numbers. The result is an estimate of what the actual product should be.

Example: Multiply 7,345 by 497
Round the numbers: 7,000 ( 500)
Multiply: 3,500,000
Actual Product: 7,345(497) = 3,650,465

Estimating tells you whether or not your answer is “in the ballpark.”

What Properties Does Multiplication Have?

If you recall, Multiplication is repeated addition. Multiplication like addition is both Commutative and Associative.

Commutative: $7 \cdot 8 = 8 \cdot 7$

Associative: $3 \cdot (4 \cdot 5) = 3 \cdot (4 \cdot 5)$

Addition has an Identity, it’s zero. Multiplication has an Identity. It’s One. If you multiply any number by 1 you don’t change it’s value.

Identity (One): $8 \cdot 1 = 8$

What Are Primes And Composites?

To determine if a number is prime or composite, follow these steps:

1. Find all factors of the number.
2. A number is prime if the number has only two factors, 1 and itself.
3. A number is composite if the number has more than 2 factors.

Here are some of the first the counting numbers broken into Primes (P) and Composites (C)
P= {2,3,5,7,11,13,17,19, ... }
C= {4,6,8,9,10,12,14,15,16, ...}
Lesson

How Do I Find The Area Of A Rectangle?

Use the formula Area=LW, to find the area of a rectangle.

A = LW
A = (3)(5)
A = 15 square feet

Which side is the length which side is the width?
The answer that is doesn’t matter, because Multiplication is Commutative.
(3)(5)=(5)(3)

Note that the answer includes the word square. This is because when we find the area we are looking for the amount of squares inside.

A rectangle has 4 sides, but we only use 2 of them to find the Area (A)

Example #1

How many one foot square tiles are needed to cover a floor that is 13 feet wide and 20 feet long?

A salesman might say write 13 x 20. The word by and the symbol ‘x’ is a reminder that for area you multiply.

A = LW
A = (13)(20) = 260 square feet

Since each tile is one square foot, 260 tiles will be needed. It’s always a

Example #2

The label on a can of paint says that it will 50 square feet. The wall that will be painted is 8 feet high and 20 cover about feet wide. How many cans will be needed?

A = LWA = 8(20) = 160 square feet.

Each can covers 50 square feet. 3 Cans would cover 150 square feet. That’s not enough. We’ll need 4 cans. Not to worry, because it’s a good idea to have extra paint. Why?

Example #3

A rectangular garden 24 feet by 40 feet is being constructed.

a. How many square feet of sod (grass) will be needed?

b. How many feet of fence will be needed to enclose the garden?

Solution.

a. A=LB A=24(40)=960 sq ft.

b. P=2L+2W
P = 2(24)+2(40)
P = 48+80
P = 128 feet

Review

What is the Perimeter of the rectangle?

P=2L+2W
P=2(3)+2(5)
P=6+10 =16 feet (no squares it’s the distance around.)
Let's see. We've discussed Addition and Subtraction, the Commutative and Associative Properties. Zero is the identity for both addition and subtraction, OK...

Let's move on! Now we're ready to continue with multiplication and division. But before that, a little *nomenclature* is in order.

8 x 7 = 56

Who's Norman Clature?

WHAT'S THAT?

NOMEN! NOMENCLATURE!

Correctamundo! This is some of the "nomenclature" we use in multiplication and division.

**MULTIPLICATION**
- Factors
- Product
- 8 x 7 = 56

**DIVISION**
- Dividend (Numerator)
- Divisor (Denominator)
- Quotient

In this example, the 8 and 7 are *FACTORS* and the result, 56, is called the *PRODUCT*.

Dictionary? I thought this was math!

Dictionary? No one has a dictionary!

Right here Professor Weissman! "A set of names or terms as those used in a particular science or art."

In this example, the 8 and 7 are *FACTORS* and the result, 56, is called the *PRODUCT*.

Division is usually indicated by a fraction, either with the bar or a slash. In division, we get a QUOTIENT when we divide a DIVIDEND (NUMERATOR) by a DIVISOR (DENOMINATOR).

What's that you say... "we get a quotient when we divide a dividend by a divisor?" Geez... and I thought "she sells seashells by the sea shore" was a tough one!
I wouldn't even try to say that! My uppers might fall out!

Because we use the X in equations?

YES! In Algebra we use letters...
  a, b, c...
  x, y, z

Oh great...now I'm back in nursery school! What's next Professor...finger painting or nappy time?

We'll cross that bridge when we come to it. Get it?

Bridge? (Har Har)

Sometimes I just crack myself up!

Getting back to our lesson, in Algebra we could have a real problem with the X when we write 8x7=56.

Can anybody explain why?

Allow me to explain. Since we might use X instead of a number, we can't use the letter X when we multiply.

In Algebra, we use the parentheses for multiplication. See!

8(7)=56
(8)7=56
(8)(7)=56

Factors Product

And sometimes, we don't need any symbols. For instance...

4(b)=4b
4·b=4b

Occasionally, we can even use a dot to represent multiplication.

8·7=56

Just make sure the dot is raised a bit so it won't be confused with a decimal point.
OK class...let's move on. We've covered the four basic operations. Now it's time to discuss powers and exponents.

**Addition**

**Subtraction**

**Multiplication**

**Division**

**Powers/Exponents**

Some terminology we'll use are as follows:

- **Exponent**
- **Base**

Go no further Prof! The answer is 12!

Well...you're thinking three 4's...that's good...but you're **adding** them.

No...I didn't...I multiplied! $3 \times 4 = 12$.

**Multiplication** is repeated addition! $3 \times 4 = 4 + 4 + 4 = 12$ but exponents mean repeated multiplication. Let me show you.

By definition, the exponent, 3, tells us to write the base, 4, as a **factor** three times.

**Factors?** I remember. So we're multiplying the three 4's. Let me give it a shot!

O'xon now...$16 \times 4 = 64$!

Oh...oh yeah! OK, so therefore $4^2 = 84$.

$4^3 = 4 \times 4 \times 4 = 4 \times 16 = 64$.

OK...let's see! $4 \times 4 = 16$. Then, $16 \times 4 = \ldots$. uh...uh...anybody got a calculator?
Right! Let's try another problem.

3^4

That's 64! We just did this one!

Correct! Here we have 4 factors of the base 3!

3^4 = 3 · 3 · 3 · 3

No! Look! The Professor switched the base and the exponent.

Alright...so we have 3x3=9. Then 9x3=27 and finally 27x3=81. Right?

Yes! But it might be easier to say 9x9=81. See what I mean?

3^4 = 3 · 3 · 3 · 3

9 · 9

81

With 10 as a base, the exponent tells us "how many zeros." So 10 with an exponent of 4 means a 1 followed by 4 zeros.

10^4 = 10 · 10 · 10 · 10

100 · 100

10000

Man...I finally got one right and I'm still wrong!

In that case, what would 10^6 mean?

10^6 = 1000000

I got that one Prof!

A 1 followed by 6 zeros. Hey, I just made my first million!

Right! Study the Powers of Ten Table a moment. Do you notice a pattern? See how a 1 is always followed by the number of zeros that the exponent indicates.

10^1 = 10
10^2 = 100
10^3 = 1000
10^4 = 10000
10^5 = 100000
10^6 = 1000000
Let me generalize a bit. Here the exponent is 5. We read this as "x to the fifth power."

\[ x^5 = x \times x \times x \times x \times x \]

In this example the base, \( x \), is written as a factor 5 times.

**Table of Squares**

<table>
<thead>
<tr>
<th>( n )</th>
<th>( n^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
</tr>
<tr>
<td>4</td>
<td>16</td>
</tr>
<tr>
<td>5</td>
<td>25</td>
</tr>
</tbody>
</table>

If we replace \( x \) with 0, 1, 2, 3, 4, and 5 we get the first 6 squares.

**Table of Cubes**

<table>
<thead>
<tr>
<th>( n )</th>
<th>( n^3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>27</td>
</tr>
</tbody>
</table>

Correct! You see there was nothing to be afraid of. Now let's move to the Table of Cubes.

**Table of Cubes**

<table>
<thead>
<tr>
<th>( n )</th>
<th>( n^3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>27</td>
</tr>
</tbody>
</table>

That must be next to the Table of Beverages. (Nyuk)
**Exponents are simply a convenient way of writing very large or very small numbers. You might not be dealing with those kinds of numbers now but who knows? For instance...**

Cogsworth ... you idiot! The next time I ask you to calculate earth's distance to the Andromeda Galaxy ... a simple $10^{10}$ will suffice!

Wrong. She doesn't weigh 21 pounds.

Geez Dr. Powers, I just love your new scale. It says I only weigh a mere 73 pounds. Incidentally, the 3 on your scale is kinda small and raised.

Exponents are called powers because they can change a small number (the base) to a much larger number very quickly. Never multiply the base with the exponent. Always rewrite the problem using factors.

I'll fix that, Professor.

That's more like it. Seven cube or seven cubed or seven to the third power. Any way you say it, is a hefty 343.
Exercise Set 3

1. Multiplication
   a. (8)(765)
   b. 64(809)
   c. 707(8)
   d. 800(0)
   e. 56(100)
   f. 56(1000)
   g. 768(1000)
   h. 60(700)
   i. 78(567)(0)(888)
   j. 9●8
   k. What is the product of 4 and 5?
   l. What is twice 15?
   m. Write the product of x and y

2. Estimate each product then find the exact answer
   a. 7,854(38)
   b. 39,804(82)

3. Translate
   a. The product of 5 and a
   b. The square of 8
   c. The cube of 2
   d. The fifth power of 10

4. Evaluate the expression for the given values.
   a. xy when x=8, y=9
   b. 7x when x=5
   c. 5xy when x=4, y=3
   d. xyz when x=2, y=5, z=10

5. Name 3 properties of multiplication that start with the letters CAI.

6. Identify the property.

7. Complete using a property of multiplication then name the property.
   a. 7=7●___
   b. 8●___=9●8
   c. 66●____=0
   d. (6●8)●11=6●_______
   e. ab=_____

8. Solutions to equations
   a. Is 7 a solution to the equation 6x=54?
   b. Is 5 a solution to the equation 30=5y

9. Write in exponential form.
   a. 5●5●5
   b. 1●1●1●1●1
   c. a●a●a
   d. xxxx
   e. 7●7●2●2●2
   f. 10●10●10●10
   g. ☺ ● ☺ ● ☺

10. Write in expanded form and evaluate.
    a. 25
    b. 105
    c. 52●25
    d. 05
    e. 15

11. Evaluate the expression for the given values.
    a. y3 y=7
    b. y5 y=2
    c. y6 y=10
    d. a=3 b=2

12. Geometry
    a. What is the formula for the area of a rectangle with sides L and W?
    b. What is the area of a rectangle with sides 5 inches and 7 inches?
    c. What is the perimeter of a rectangle with sides 5 inches and 7 inches?
    d. What is the area of a square with a side 5 inches?

13. Find all the factors of:
    a. 4
    b. 8
    c. 12
    d. 16
    e. 24
    f. 36
    g. 48
    h. 100

14. Break each number into its prime factors:
    a. 4
    b. 8
    c. 12
    d. 16
    e. 24
    f. 48
    g. 100

15a. Find all possible pairs of factors whose product is 12 then list those whose:
    a. sum is 7
    b. difference is 11
    c. sum is 8
    d. difference is 4

16a. Find all possible pairs of factors whose product is 18 then list those whose:
    a. sum is 11
    b. difference is 17
    c. sum is 9
    d. difference is 7
Free Software For All Mathematics Subjects Including Statistics

Jokes Set #3

Some engineers are trying to measure the height of a flag pole. They only have a measuring tape and are quite frustrated trying to keep the tape along the pole: It falls down all the time. A mathematician comes along and asks what they are doing. They explain it to him. "Well, that's easy..."

Brain Teasers Set #3

Three men go to a cheap motel, and the desk clerk charges them a sum of $30.00 for the night. The three of them split the cost ten dollars each. Later the manager comes over and tells the desk clerk that he overcharged the men, since the actual cost should have been $25.00. The manager gives the bellboy $5.00 and tells him to give it to the men. The bellboy, however, decides to cheat the men and pockets $2.00, giving each of the men only one dollar.

Now each man has paid $9.00 to stay for the night, and $3 x $9.00 = $27.00. The bellboy has pocketed $2.00. But $27.00 + $2.00 = $29.00. Where is the missing $1.00?
Answers to Exercise Set 3

1a. 6,120
b. 51,776
c. 5,656
d. 560
e. 5600
f. 56,000
g. 768,000
h. 42,000

i. 0
j. 72
k. 20
l. 30
m. xy

2a. 320,000 ; 298,452
b. 3,200,000 ; 3,263,928

c. 82
a. No
b. No.
c. 105
9a. 53

2a. 320,000 ; 298,452
b. 3,200,000 ; 3,263,928

3a. 5a
a. 82
b. 23
c. 105
9a. 53

Brain Teaser #3 Answer

it's all in how you phrase the question.

The men paid $27 total. $25 to the manager, and $2 to the bellboy.
The fact that they also originally paid an extra $3 and then got it back again is entirely irrelevant.

What makes this puzzle seem so impossible?? it's the question they ask you at the end.
There is no missing dollar. It's a trick, a math trick. here's the answer.
the three men:

$30 (what they paid in the beginning)
-$5 (amount deducted)
+$3 (amount given back)

=$28

$28
+$2 (amount bellboy kept)

=$30

there's no missing dollar!!!!

The men paid $27
The motel got $25
The bellboy got $2

$25 + $2 = $27